Application of the Complex Integration Path in the Theory of Leaky Modes

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Abstract— **A variational expression for the complex core parameter is used to check the validity of the concept of complex integration path in the theory of leaky modes. An numerical example is presented to show the accuracy of the concept.**

I. INTRODUCTION

THE calculation of the fields and the bending losses of leaky modes in bent optical waveguides leads to integrals over an infinite cross section [1], [2]. These integrals do not converge, because the fields increase to infinity far away from the fiber axis. This difficulty can be overcome by the concept of the complex integration path. This concept is used in the theory of wave propagation, for slab and circular optical waveguide, in [3]–[7]. Until now, numerical investigations were only made for the slab waveguide [4], [5]. In this letter we show for the first time numerically that the concept can also be used for circular optical waveguides. Moreover, it will be shown that the theory of truncated leaky modes yields wrong results [7]. To the best of our knowledge, no numerical comparison has been made in literature between the concept of truncated leaky modes and the concept of complex integration path.

Furthermore, in this letter we investigate the leaky loss of the LP_{11} mode of a straight optical waveguide that propagates at a wavelength greater than the theoretical cut-off wavelength, for instance in a depressed cladding (DC) fiber. Those modes are leaky even if the fiber is straight.

II. THEORY

The variational expression for the complex core parameter U is given by [7]

$$U^2 = \frac{\int_{R_c} \left\{ \left(\frac{dF_g}{dR} \right)^2 + \left[\frac{l^2}{R^2} + V^2 f(R) \right] F_g^2 \right\} R\, dR}{\int_{R_c} F_g^2 R\, dR} \tag{1}$$

where U is defined by

$$\begin{aligned} U &= a_n \sqrt{k_0^2 n_m^2 + \gamma^{(0)2}} \\ &= \sqrt{V^2 + Q^2} \end{aligned} \tag{2}$$

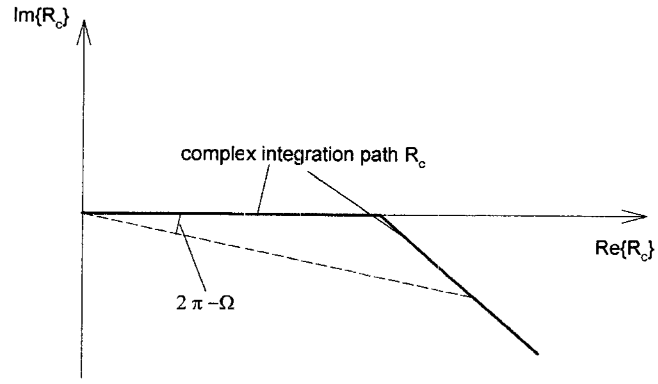

Fig. 1. Path of integration in the complex R plane.

with the complex propagation coefficient $\gamma^{(0)} = \alpha' + j\beta$, $R = \rho/a_n$ with the core radius a_n (compare Fig. 2), the maximum refractive index n_m, the real normalized frequency V, and the complex cladding parameter $Q = Q_r + jQ_i$. R_c is the complex integration path. The other parameters have the usual meanings [7]. F_g is the radial field amplitude distribution of the leaky mode in a circular optical waveguide. The field in the cladding is described by a Hankel function of the second kind. For reell $R \to \infty$, we have

$$\lim_{R\to\infty} |H_\ell^{(2)}(QR)| \to \begin{cases} \infty & \text{for, } \operatorname{Im}\{QR\} > 0 \\ 0 & \text{for, } \operatorname{Im}\{QR\} < 0 \end{cases} \tag{3}$$

and thus the integrals in (1) do not converge for a real integration path $\operatorname{Im}\{R_c\} = 0$ (Fig. 1), because for leaky modes we have $Q_i > 0$, $Q_r > 0$. The approach taken in [3]–[6] to overcome this problem is simply to deform the path of integration into the complex $R = R_r + jR_i$ plane in such a way that for large $|R|$ the decay due to the imaginary part of R compensates for the growth caused by the negative imaginary part of Q. The integral converges only if

$$\operatorname{Im}\{(Q_r + jQ_i)(R_r + jR_i)\} < 0 \tag{4}$$

and with $\tan\Omega = R_i/R_r$ we get

$$Q_i + Q_r \tan\Omega < 0. \tag{5}$$

For numerical $Q_i < 0$ and $Q_r = 0$ or $\Omega = 0$ we have the relation for guided modes. For leaky modes we have $Q_i > 0$, $Q_r > 0$, and $R_r > 0$ we get

$$-\frac{Q_i}{Q_r} < \tan\Omega < 0. \tag{6}$$

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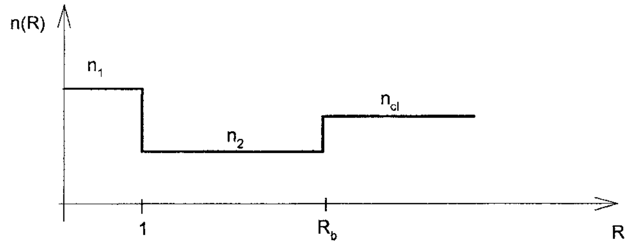

Fig. 2. Refractive index profile of a DC fiber.

If Ω lies in this interval its exact value is unimportant, because the integrals in (1) do converge for all values in this interval.

III. Numerical Results and Discussion

For numerical simulations we used the profile parameters $a_n = 4.5\,\mu$m, $R_b = 7$, $n_1 = 1.461134$, $n_2 = 1.455730$ and $n_{cl} = 1.45733$ of a typical DC fiber [8] (Fig. 2). We calculated the leaky loss for the LP_{11} mode whose theoretical cut-off wavelength is 1238.8 nm for $R_b = 1$ (MC fiber) and 1045.8 nm for $R_b = 7$ (DC fiber). In Fig. 3 we compare the leaky loss coefficient $\alpha = 8.68\alpha'$ of the LP_{11} mode for $R_b = 1$ (MC fiber) and in Fig. 4 for $R_b = 7$ as a function of wavelength. There are no differences between the exact (labeled as "exact") calculation of the leaky loss by solution of the complex eigenvalue equation derived with the multilayer method and the use of the complex integration path method (labeled as "complex integration path") based upon (1). The dashed curves (labeled as "$R_m = 7$") show the results for the theory of truncated leaky modes. If the upper limit of integration in (1) is $R = R_b = R_m$ instead of infinity, the calculated loss deviates significant from the exact loss. This means that the concept of truncated leaky modes [6] is not adequate for calculating the integrals in (1). Further numerical results are included in [1].

IV. Conclusion

We have shown numerically that the concept of the complex integration path can be used to calculate integrals that occur in the well-known variational expression for the complex core parameter of leaky modes. The complex integration path method can be used in all fields where leaky modes are important.

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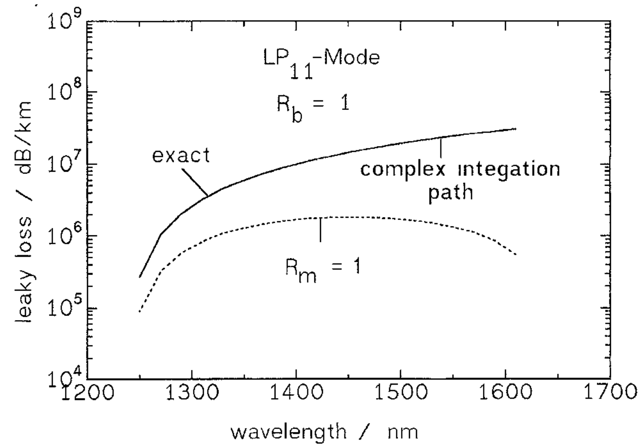

Fig. 3. Leaky loss of the LP_{11} mode as a function of wavelength: $R_b = 1$.

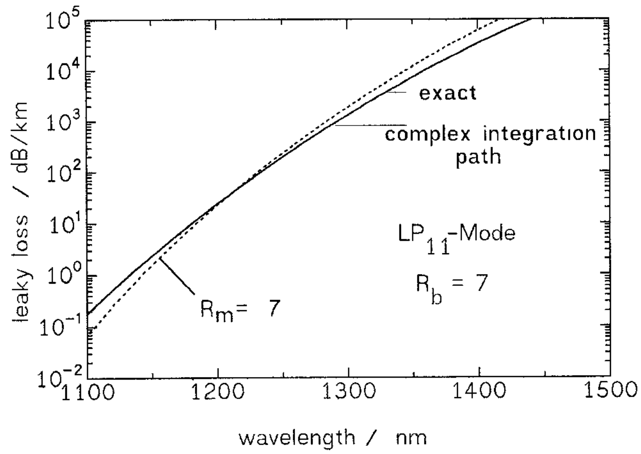

Fig. 4. Leaky loss of the LP_{11} mode as a function of wavelength: $R_b = 7$.

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